Universität Potsdam
Institut für Mathematik
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Stieltjes integration and iterated integrals

Exercise Sheet 1

Discussion Wednesday April 26

Exercise 1. (1) Let $\mathcal{R}$ be a set theoretic ring. Show that $A \cap B \in \mathcal{R}$ for all $A, B \in \mathcal{R}$.

(2) Show that there is a set theoretic ring $\mathcal{R}$ over a set Ω and an $A \in \mathcal{R}$ such that $\Omega \setminus A \notin \mathcal{R}$.

Exercise 2. Let $\mu : \Omega \to \hat{\mathbb{R}}$ be a signed measure and (P, Q) a Hahn decomposition of Ω corresponding to μ.

(1) Show that

$$\mu^+ = \min_{\substack{\mu_1,\mu_2 \in \mathcal{M}_{\geq 0}(\Omega): \\ \mu_1 - \mu_2 = \mu}} \mu_1, \quad \mu^- = \min_{\substack{\mu_1,\mu_2 \in \mathcal{M}_{\geq 0}(\Omega): \\ \mu_1 - \mu_2 = \mu}} \mu_2, \quad \mu^+ + \mu^- = \min_{\substack{\mu_1,\mu_2 \in \mathcal{M}_{\geq 0}(\Omega): \\ \mu_1 - \mu_2 = \mu}} (\mu_1 + \mu_2)$$

(2) Show that (μ^+, μ^-) is the unique decomposition $\mu = \mu^+ - \mu^-$ into nonnegative measures such that $\mu^+ \perp \mu^-$.

(3) Show that (μ^+, μ^-) is the unique decomposition $\mu = \mu^+ - \mu^-$ into nonnegative measures such that $\mu^+(\Omega) = \sup \mu$ and $-\mu^-(\Omega) = \inf \mu$.

Exercise 3. Show that if μ_f exists, then f is locally bounded and right-continuous with left limits existing (cadlag).